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CRITICISMS AND DISCUSSIONS.

ON THE THEORY OF PROBABILITIES.

A study of philosophical literature and periodicals of the last thirty years leads to the conclusion that the theory of probability has made little progress since the time of De Morgan and Jevons. Coupled with the lack of progress there has been an enormous development in the use of statistics, and there is other evidence of a growth of interest in the practical problems of probability. A large extension in the applications of a science, without a corresponding growth of interest in its principles, is sometimes dangerous. In the particular case of the theory of probability it is clear that a serious reexamination of its principles is urgently needed. A hint of a breeze in this direction is to be seen in *Mind,* October, 1918, where Prof. C. D. Broad restates the question of the relation of probability and induction. It is, I think, worth while to make a few general remarks in the lull before Professor Broad's lead is taken up and the whole question put on a more satisfactory basis.

Probability propositions are to be distinguished by the fact that they involve certain special notions. It is necessary first of all to discuss their form in order to see the nature of the entities to which the notions of probability can significantly be applied, and then to discuss what primitive ideas the system needs. It has been recognized[1] that in all estimates of probability there is an implicit or explicit reference to a state of knowledge relative to which the probability of some proposition is to be assessed. Probability in general has reference to two propositions; the fundamental notion is then the probability of one proposition being true assuming another is true, or (as it may be more conveniently expressed) the probability of one proposition *on* another.

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[1] Cf. Jevons, *The Principles of Science,* p. 212; Broad, "The Relation between Probability and Induction," *Mind,* Oct., 1918.

The propositions consist in a relation *less than, greater than* or *equal to* between probabilities, or in the evaluation of the probability of a certain proposition on a certain ground. It will be important to discuss the relation between these notions. We may, e. g., tentatively suggest that it may be necessary to take only one of these notions, *the probability of, more probable than, less probable than, as probable as,* as a primitive idea in the system. For we may use the relation *more probable than* as primitive, defining *less probable than* as its converse and *as probable as* as the negation of the sum of these relations. (This is only satisfactory if to say that one entity is more probable than another, if it is neither more nor less probable than the other entity, is to give a permissible interpretation of the notions involved.) Then the probability of an entity will be defined as the class of equally probable entities. The problems of the nature of entities to which these notions can significantly be applied, and of the relation between them, complete the first division of the theory.

A very superficial examination of the structure of our belief will convince us that there must be some probability propositions which are to be accepted because of their relation to other propositions involving the same notions, and some which are to be accepted independently of any such proposition. This division into primitive and derived probability propositions is essential. It must, however, be noted that primitive probability propositions may very well be logically derivative. The property which distinguishes them from other propositions of the group is that they are not derived from any other *probability* propositions. But it is a difficult matter to effect this division. The possibility of a theory of probability depends upon the fact that there is a certain measure of agreement as to which probability propositions are true and which are not. When, however, we try to discover which of these propositions are accepted derivatively and which are accepted primitively, we find that there is no great consensus of opinion. This science, like, for example, any serious science of ethics, must consist in part of making definite certain beliefs vaguely held by a number of people. It seems to be the case that people believe vaguely and with varying degrees of conviction large numbers of propositions not logically independent. A logical treatment of probability propositions which is necessary if a system is to be established, will then include a discussion as to which are to be taken as primitive in the system and which are to be taken as derivative. The procedure will consist in discovering which various subsets of the propositions of probability are sufficient to imply the rest. It will then be necessary to choose one of these subsets; in our choice we shall pay due attention to the varying degrees of conviction with which the various propositions are believed. A careful consideration of the propositions to be taken as primitive in the theory of probability will throw light on the significance of the notions involved, and may perhaps lead to a definition of *more* or *less probable.* The existence of such a definition would then enable us to decide as to the truth of any other primitive probability propositions. It seems plausible to suppose that such a definition, however, will only come from a consideration of the generally accepted propositions of probability.

When the question of the primitive propositions has been discussed, an investigation into "probability inference" must be attempted. We must have, as in geometry, certain postulates as to how some probability estimates can be made to yield others. The general problem is to find the relation between the probability of p say on q in terms of the probabilities of a on b, c on d....To hold that probability inference is a possibility is to hold that certain probabilities determine certain other probabilities. The question as to what particular probabilities must be given in order that certain other probabilities may be determined will have to be discussed. There is, however, one general aspect of the problem upon which I wish to make a remark. Clearly we shall not expect to get a relation between the probabilities of a on b, c on d, e on f....and p on q, if there is no relation between a, b, c, d, e, f and p and q. This delimits at once the province it is possible for probability inference to cover. The fundamental logical relations between propositions are very few: out of them, all other logical relations of propositions can be manufactured. The relations are included in the following ones: (1) A proposition may be the contradictory of the other; (2) a proposition may be the sum of a pair of propositions; (3) a proposition may be the product of a pair of propositions; (4) a proposition may imply another proposition.[2] If, then, we are able to discover relations which it is plausible to suppose subsist between the probabilities of a on b, c on d, e on $f \ldots . p$ on q when p and q stand in any of these fundamental logical relations to $a, b, c, d, e, f \ldots .$ careful manipulation will enable us to get relations between probabilities of p on q and the probabilities of a on b, c on $d, \ldots .,$ when p and q stand in any logical relation whatever to $a, b, c, d \ldots .$ The role of the postulates to be adopted is plain. We want relations between the probability of p on q, *not-p* on q, *p or r* on q, *p and r* on q; relations between the probability of p on q and r on q when p implies r; and further, relations between probabilities in which the state of knowledge relative to which they are to be assessed is varied in an analogous manner. This at once delimits the field: the fundamental principles giving the way in which probabilities can be combined will be correspondingly restricted. The form of these postulates should be noticed. One used by Jevons in *The Principles of Science* (p. 205), e. g., runs as follows. *The sum of the probabilities of two propositions is equal to the probability of the sum of the propositions together with the probability of the product of the propositions.* When we are concerned with getting further rules for the combination of probabilities we treat the postulates as propositions defining the way in which probabilities can be combined. As an example of the form of the results we may take: any predicate ϕ satisfying the propositions

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[2] It is not necessary to discuss here, whether the definition of implication in terms of the first two propositions is possible. (Cf. the definitions of Whitehead and Russell.)

$$\phi(x, h) + \phi(y, h) = \phi(x \text{ and } y, h) + \phi(x \text{ or } y, h)\,;$$
$$\phi(x, h) \times \phi(y, x \text{ and } h) = \phi(x \text{ and } y, h)$$

also satisfies the proposition

$$\phi(x, h) + \phi(y, h) + \phi(z, h) - \phi(y \text{ and } z, h) - \phi(z \text{ and } x, h) - \phi(x \text{ and } y, h) + \phi(x \text{ and } y \text{ and } z, h) = \phi(x \text{ or } y \text{ or } z, h).$$

The interesting point is that, any ϕ whatever satisfying the postulates satisfies the derived propositions. The process of obtaining the consequences of the postulates adopted is an exercise in pure logic and like the theory of the ancestral relation, which is vital for the analysis of mathematical induction or the theory of vector families which is important in the analysis of the notion of quantity, forms a chapter in the theory of modern logic. The process of deduction is then identical with the processes used in any branch of pure mathematics as, e. g., in projective geometry. The antithesis traditionally drawn between inductive inference and deductive inference is therefore to be abandoned, if a view of probability theory in accordance with modern logic is to be developed. As in all other branches of knowledge in which there are some primitive and some derived propositions, the truth or falsehood of the derived propositions rests on that of the primitive propositions and on the truth of the postulates adopted as to the behavior of the concepts involved in the propositions, if we may assume that the purely logical problem of developing the consequences of the postulates has been correctly solved. If in any group among the derived propositions there are some which are exceedingly unplausible, the postulates and the primitive propositions adopted must be examined and modifications may then be made.

But it is important to keep the purely logical part of the subject separate from the inquiry into the truth of primitive probability propositions. The analogy with geometry is particularly suggestive at this point. Pure geometry consists in the investigation of the logical consequences of different systems of postulates, and in the analysis of these postulates. Applied geometry examines what propositions are true of the external world and takes over from pure geometry the logical consequences of the propositions. And it is in this connection that it is important to point out that the derivation of the consequences of the postulates adopted in probability theory can be put in the form: the property ϕ satisfies certain propositions, therefore it satisfies certain other propositions. We will take one example. It has been assumed in the traditional treatment of the subject that probabilities are quantitative. De Morgan, for example, in his *Essay on Probabilities* (p. 18), says in answer to objections as to the measurement of probability: "Again as we are most unquestionably justified in saying that it is more probable we shall draw one of the two *a* or *b*, than that we shall draw *a*, the argument must be directed against the method of measurement, not against the possibility of a measure: for wherever *more or less* are applicable terms, twice, thrice, etc., must also be conceived to be possible whether we can ascertain how to find them or not." We postulate that probabilities are quantitative in order to satisfy a vague unanalyzed feeling about them; this feeling would perhaps be as readily satisfied by the postulate that probabilities form a series (i. e., by the postulate that the relation *less probable than* is nonreflexive, connected and transitive) or possibly by a part of this postulate.[3]

This question of probabilities being quantitative is only one of the various problems as to which postulates most nearly represent our feelings about probabilities. In the light of modern logical analysis many assumptions as to the nature of probability no longer seem obviously necessary or satisfactory. It is a matter of great importance for the development of the theory of probability on a secure logical basis that these assumptions should be carefully analyzed into their constituent parts.

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[3] In particular it is possible that it may not be necessary to postulate connexity.

The object of this paper may then be summed up in the following way. Before embarking upon the general reconsideration of the interesting and important problems of probability, it is worth while to consider in general terms the various parts of the theory, with a view to laying down the lines which any satisfactory theory must follow. The form of the present inquiry is then sufficiently explained. Probability theory consists of a set of propositions grouped together by reason of the concepts involved in them. A satisfactory treatment of the subject should give a discussion of the nature of these notions, and in particular a discussion of the nature of the entities to which they can significantly be applied. It should also give in their most fundamental form postulates involving relations between various probabilities, the propositions implicated in these probabilities being connected by various fundamental logical relations. Further, the logical implications of these postulates should be developed, so that the relations between probabilities involving propositions connected by complicated logical relations may be formulated. The postulates and their logical consequences provide the principles of probability inference. In order that these laws of inference may be applied to the problems of science, it is further necessary to examine the primitive probability propositions which are to be assumed, and to work out the results logically entailed.

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THE FUNCTION OF GENERALIZATION.

I have a friend in one of the older sciences who has a reputation on both sides of the Atlantic for some splendid work he has done in one corner of his field. He is in many respects a broad-minded, tolerant man, especially when he is dealing with his friends or is concerned with matters outside of his province of technical investi-